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ATMOSPHERIC LIMITS ON THE OBSERVATIONAL CAPABILITIES OF AEROSPACECRAFT

by John C. Evvard

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Cleveland, Ohio

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ABSTRACT

Atmospheric turbulence decreases the ground observational capability of an aerospacecraft. Most of the effect arises from transient distortions of the light path near the surface (up to 15 or 20 km) where the air density is high. Hence, the viewing accuracy of a satellite observer looking at the ground is generally much higher than for a ground observer viewing a satellite. Estimates of the atmospheric resolving-power error for both the ground and the aerospacecraft observer have been made. The estimated uncertainty of viewing a point on the ground directly beneath an aerospacecraft need be no larger than 12 centimeters. Expressions are derived for the altitude and angular dependency of the uncertainty.

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SUMMARY

Atmospheric turbulence decreases the ground observational capability of an aerospacecraft. Most of the effect arises from transient distortions of the light path near the surface (up to 15 or 20 km) where the air density is high. Hence, the viewing accuracy of a satellite observer looking at the ground is generally much higher than for a ground observer viewing a satellite.

The ratios of these positional uncertainties have been estimated by assuming plausible or limiting relations for the instantaneous density gradients in the statistically fluctuating atmosphere. Combinations of this information and the observed angular position uncertainty of a twinkling star then yield an estimate of the atmospheric resolving-power error for both the ground and the aerospacecraft observers. The estimated uncertainty of viewing a point on the ground directly beneath an aerospacecraft need be no larger than 12 centimeters. Expressions are derived for the altitude and angular dependency of the uncertainty.

INTRODUCTION

Limits on the observational capabilities of an observer or camera stationed on an aerospacecraft are largely determined either by the image brightness contrast or by the optical resolving power. The image brightness contrast is initially limited by diversity at the object, but reductions in contrast may result because of intervening clouds, dust, smoke, and aerosols in the transient atmosphere (refs. 1 and 2). The optical resolving power is limited either by the optical instrument quality (perfection) and size (diffraction limit) or by distortion in the light-ray path due to transient density gradients arising from atmospheric turbulence and temperature gradients (refs. 3 to 5). This report is concerned with the limits due to atmospheric turbulence on the resolution with which an

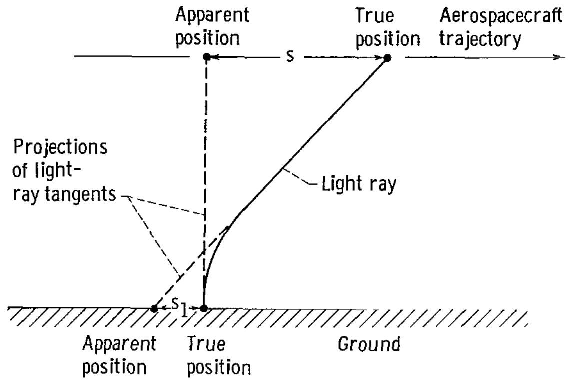

Figure 1. - Apparent image position for ground and aerospacecraft observers.

aerospacecraft observer might view the ground. Related studies are reported in references 5 to 9.

The twinkling of stars is a well-recognized consequence of atmospheric density gradients (ref. 4). The positional uncertainty of a twinkling star is generally no more than 3 seconds of arc, as observed from sea level, and may occasionally be only a few tenths of a second. The corresponding fluctuation in the apparent position of an aerospacecraft, as viewed from the ground, can therefore be estimated as a function of flight altitude. For a satellite at 320 kilometers, the uncertainty is usually less than 4.7 meters.

In figure 1, two observers, one on the ground and one on an aerospacecraft, are looking at each other. Each appears to see the other along the projected tangent to the local light path. The light-path distortion, however, is large near the ground, where the air density is high, and is small near the aerospacecraft. Correspondingly, the position error in observing the aerospacecraft should be larger than the position error in observing the ground. Also, the error in observing the aerospacecraft increases with altitude, whereas the error in observing the ground is nearly independent of altitude if the aerospacecraft is essentially above the bulk of the atmosphere.

Because both are using the same instantaneous light path, the ratio of observational errors s/s_1 of the ground and aerospacecraft observers may be estimated by assuming instantaneous lateral density gradients with exponential altitude decay. Utilization of the statistical angular positional uncertainty of a twinkling star then yields an estimate of the atmospheric-optical-resolving-power error for both the ground and the aerospacecraft observers. This is an extension of an earlier study (ref. 5) which only considered a vertical light ray. The results of reference 5 will also be condensed herein employing slightly different assumptions for evaluation of the constants.

SYMBOLS

a	lateral wavelength (x-direction)
b	parallel wavelength (y-direction)
$(\partial f/\partial x)_m$	constant controlling amplitude of sinusoidal density variation, eq. (15)
$(\partial f/\partial x)_0$	constant related to density gradient normal to light path near $x = 0$
$g(R)$	function of R defined by eq. (37)
k	proportionality constant in eq. (2) of value 0.0003
n	index of refraction
R	radius vector from earth's center
R_0	value of R at earth's surface
r	radius of curvature of light ray
s	position uncertainty in orbit tangent plane of aerospacecraft as viewed from sea level
s'	position uncertainty of aerospacecraft in direction normal to light path as viewed from sea level
s_1	position uncertainty of point on earth's surface as viewed from aerospacecraft
s_1'	position uncertainty as viewed from aerospacecraft of point at sea level in direction normal to light path
x	coordinate perpendicular to light-ray path
y	coordinate along light-ray path
y_0	constant in eq. (3) of approximate value 8 km
α	constant defined as $2\pi y_0/b$
δ	normal distance from nonturbulent light path
θ	angle that light ray tangent makes to zenith
θ_0	value of θ at earth's surface, $\theta_0 = \psi_0$
θ_1	value of θ along light path without turbulence
θ_2	perturbation in θ due to turbulence, eq. (31)
ρ	density of air
ρ_{sl}	density of air at sea level

φ coordinate angle at earth's center that radius vector R makes to zenith line (fig. 4)

φ_0 phase angle, eq. (15)

ψ angle light path makes to radius vector from earth's center (fig. 4)

ψ_0 value of ψ at earth's surface, $\psi_0 = \theta_0$

ψ_1 value of ψ along light path without turbulence

ψ_2 perturbation in ψ due to turbulence, eq. (31) (note that $\psi_2 = \theta_2$)

DERIVATIONS

A plane wave of light passing through a medium of variable index of refraction will follow a curved path. The bending of the light ray is toward the portions of the medium having the larger index of refraction. In the atmosphere, the radius of curvature r of the path is given by the well-known relation

$$\frac{1}{r} = -\frac{1}{n}\frac{\partial n}{\partial r} \approx -\frac{\partial n}{\partial r} \tag{1}$$

where $\partial n/\partial r$ is the component of the gradient normal to the light-path direction. The index of refraction n is related to air density by the relation

$$n - 1 = k\frac{\rho}{\rho_{sl}} \tag{2}$$

where k has the approximate value 0.0003 and ρ/ρ_{sl} is the ratio of local to sea level density. Without turbulence, the density ratio may be approximated by an exponential decay with altitude y so that equation (2) becomes

$$n - 1 = ke^{-y/y_0} \tag{3}$$

where y_0 has the approximate value of 8 kilometers corresponding to a decrease in atmospheric density by a factor of 2 for each 5.5-kilometer (18 000-ft) increase in altitude.

In addition, there will be random local changes in the index of refraction due to turbulence, composition, and temperature gradients that may lead to bending of the light-ray path. This transient bending causes the twinkling of stars and the loss of optical resolution. No bending occurs, however, where there is no air. Hence, any assumed relation for the lateral change of index of refraction with altitude due to all causes including turbulence must contain a weighting factor resembling the exponential term e^{-y/y_0} of equation (3).

Vertical Light-Ray Path

The vertical light ray is the simplest case because without turbulence the unperturbed path is a straight line. Two cases are studied. The first assumes an instantaneous constant lateral density gradient with an exponential altitude decay. The second, also with an exponential altitude decay, is a constant-wavelength sinusoid. From these two cases, a plausible ratio is obtained for the observational errors of reciprocally viewing ground and aerospacecraft observers from which the resolution of ground objects may be estimated.

Constant lateral density gradient. - If the air density at some instant is assumed to vary linearly for short distances x normal to the light path, the density ratio assumes the following form

$$\frac{\rho}{\rho_{sl}} = \left[1 + \left(\frac{\partial f}{\partial x}\right)_0 x\right] e^{-y/y_0} \tag{4}$$

where $(\partial f/\partial x)_0$ is the assumed constant density gradient normal to the light path. Hence, from equations (1) and (2) and noting that $\partial r = -\partial x$

$$\frac{1}{r} = k\left(\frac{\partial f}{\partial x}\right)_0 e^{-y/y_0} \tag{5}$$

The apparent and true positions of the ground and aerospacecraft observers are shown in figure 2. The angle θ may be determined from the relation

$$d\theta = \frac{dy}{r \cos \theta} \approx \frac{dy}{r} = k\left(\frac{\partial f}{\partial x}\right)_0 e^{-y/y_0} dy \tag{6}$$

or

$$\theta = ky_0\left(\frac{\partial f}{\partial x}\right)_0 \left(1 - e^{-y/y_0}\right) \tag{7}$$

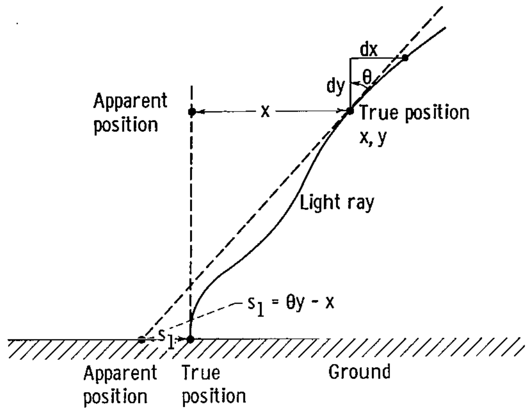

Figure 2. - Apparent and true positions of ground and aerospace observers.

But

$$\frac{dx}{dy} = \tan\theta \approx \theta \tag{8}$$

Hence, the positional uncertainty s of the satellite (fig. 1) as viewed from the ground is, by equations (8) and (7),

$$x = s = ky_0^2\left(\frac{\partial f}{\partial x}\right)_0\left[\frac{y}{y_0} - \left(1 - e^{-y/y_0}\right)\right] \tag{9}$$

But, from figure 2,

$$s_1 = \theta y - x = \theta y - s \tag{10}$$

Substitutions from equations (7) and (9) give

$$s_1 = ky_0^2\left(\frac{\partial f}{\partial x}\right)_0\left(1 - e^{-y/y_0} - \frac{y}{y_0}e^{-y/y_0}\right) \tag{11}$$

The ratio of the positional uncertainties looking up and down, from equations (9) and (11), is therefore

$$\frac{s}{s_1} = \frac{\frac{y}{y_0} - 1 + e^{-y/y_0}}{1 - e^{-y/y_0} - \frac{y}{y_0} e^{-y/y_0}} \tag{12}$$

The exponential terms of equations (9), (11), and (12) are generally negligible above about 50 kilometers. From equation (11), the accuracy of viewing the ground is essentially independent of further increases in altitude. The positional uncertainty of viewing the satellite (eq. (9)) increases linearly with altitude. The ratio of uncertainties (eq. (12)) becomes

$$\frac{s}{s_1} = \frac{y}{y_0} - 1 \tag{13}$$

This ratio does not contain the assumed density gradient constant $(\partial f/\partial x)_0$. Hence, the same result would have been obtained even if the density gradient were doubled or tripled. This fact lends credence to the use of equation (13) for estimating the uncertainty ratio. However, the light path represented by this derivation monotonically curves in the same direction away from the zenith. In the atmosphere, the curvature is random, reversing with time and altitude. Reversals tend to cancel the positional uncertainties so that the error in seeing the ground from an aerospacecraft should be less than that estimated by equation (13). (See the section <u>Constant-wavelength sinusoid.</u>)

For a satellite at an altitude of 320 kilometers, the ratio s/s_1 from equation (13) ($y_0 \approx 8$ km) is about 39. But the value of s corresponding to an angular uncertainty of 3 seconds of arc is 4.66 meters. Hence, the satellite observer should be able to view objects on the ground with an error due to turbulence of no more than about 12 centimeters. This agrees closely with the value of 10 centimeters obtained by quite a different approach (ref. 8, p. 1384). Thus, the observational capability of a satellite observer with proper optical equipment can be superb. The altitude dependence of the resolution may be estimated from equation (11) so that the random error (in cm) representing the viewing fuzziness or resolution follows the relation

$$\text{Random error} \leq 12\left(1 - e^{-y/y_0} - \frac{y}{y_0} e^{-y/y_0}\right) \tag{14}$$

The parenthetical quantity of equation (14) is plotted as a function of altitude in figure 3. From this relation and the fact that any obscuring cloud cover would lie below 30 kilometers, the conclusion may be drawn that the limiting observational capability of very high flying aircraft and satellites is essentially the same. The most sensitive altitude (maximum value of ds_1/dy) for accumulating the error is at the point where $y = y_0$ or at an altitude of about 8 kilometers.

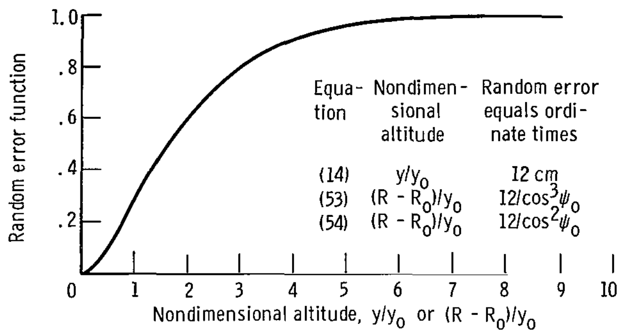

Figure 3. - Dependence of ground resolution on aerospacecraft altitude.

Constant-wavelength sinusoid. - A somewhat more sophisticated example might be obtained by assuming temporarily that the atmospheric density varies sinusoidally in both x- and y-directions as follows

$$\frac{\rho}{\rho_{sl}} = \left[1 + \frac{a}{2\pi}\left(\frac{\partial f}{\partial x}\right)_m \sin\left(\frac{2\pi x}{a} + \frac{2\pi y}{b} + \varphi_0\right)\right] e^{-y/y_0} \tag{15}$$

where $(\partial f/\partial x)_m$, a, b, and φ_0 are constants at the instant of the calculation. The constants a and b are wavelengths and φ_0 is a phase angle. From equations (1), (2), (6), (8), and (15), the following relation is obtained:

$$\frac{d\theta}{dy} = \frac{d^2x}{dy^2} = k\left(\frac{\partial f}{\partial x}\right)_m \cos\left(\frac{2\pi x}{a} + \frac{2\pi y}{b} + \varphi_0\right) e^{-y/y_0} \tag{16}$$

Solutions to this equation will be considered for wavelengths a and b of order 30 meters or larger. From equation (9), the value of x, even at 30 kilometers, is less than 1 meter. Hence, the angular shift due to the term $2\pi x/a$ may generally be neglected. Equation (16) thus becomes

$$\frac{d\theta}{dy} = \frac{d^2x}{dy^2} = k\left(\frac{\partial f}{\partial x}\right)_m \cos\left(\frac{2\pi y}{b} + \varphi_0\right)e^{-y/y_0} \tag{17}$$

The integration of this equation is included in reference 5. The results for altitudes above 50 kilometers are as follows:

$$\theta = k\left(\frac{\partial f}{\partial x}\right)_m \frac{y_0 \cos \varphi_0(1 - \alpha \tan \varphi_0)}{1 + \alpha^2} \tag{18}$$

$$s_1 = ky_0^2\left(\frac{\partial f}{\partial x}\right)_m \frac{(1 - \alpha^2) \cos \varphi_0 - 2\alpha \sin \varphi_0}{\left(1 + \alpha^2\right)^2} \tag{19}$$

$$\frac{s}{s_1} = \frac{(1 + \alpha^2)(\cos \varphi_0 - \alpha \sin \varphi_0)}{(1 - \alpha^2) \cos \varphi_0 - 2\alpha \sin \varphi_0} \frac{y}{y_0} - 1 \tag{20}$$

where

$$\alpha = \frac{2\pi y_0}{b} \tag{21}$$

Consider three cases.

(1) If the wavelength b is long, $\alpha \rightarrow 0$ and

$$\frac{s}{s_1} \rightarrow \frac{y}{y_0} - 1$$

as given by equation (13). Also,

$$s_1 = ky_0^2\left(\frac{\partial f}{\partial x}\right)_m \cos \varphi_0$$

as before (eq. (11)), except that $(\partial f/\partial x)_m \cos \varphi_0$ replaces $(\partial f/\partial x)_0$. The limit on the resolution is identical to that given by equation (14).

(2) If the wavelength b is short, α is large. For b = 44 meters, $\alpha \approx 1000$. If $\alpha \tan \varphi_0 >> 1$,

$$s_1 = -ky_0^2\left(\frac{\partial f}{\partial x}\right)_m \frac{\cos \varphi_0}{\alpha^2}$$

$$\frac{s}{s_1} = (\alpha \tan \varphi_0)\frac{y}{y_0} - 1$$

Clearly, the ground resolution s_1 is, in this case, considerably better than that estimated by equation (14).

(3) If b is short, α is large. But the phase angle φ_0 might be chosen so that $\alpha \tan \varphi_0 \approx 1$. Then, from equation (20), s/s_1 could feasibly be nearly zero. The angle θ also is fortunately near zero, but a lateral shift may occur of magnitude

$$s_1 = ky_0^2\left(\frac{\partial f}{\partial x}\right)_m\left(\frac{\cos \varphi}{1 + \alpha^2}\right)$$

The value of $\cos \varphi_0/(1 + \alpha^2)$ is very much less than 1 so that s_1 is smaller than that estimated by equation (11).

Both models (constant gradient and sinusoidal gradient) are obviously highly idealized. In the atmosphere, the turbulent processes are sufficiently random that even a damped sinusoidal density variation would be a poor approximation. Certainly the wavelength of the disturbance would vary from point to point. Also, the light-path distortion would not, in general, be limited to a single lateral direction, as was assumed. The light path might follow a randomly spiralled path up through the atmosphere with many variations in direction and curvature associated with atmospheric turbulence (refs. 6 to 10).

In the constant-density-gradient case, the values of θ, s, and s_1 increase monotonically with altitude. For the sinusoidal case, θ alternately increases and decreases with altitude, as may be inferred from equation (17). The net result from the equations is a lower positional error in viewing a point on the ground from an aerospacecraft than is given by the constant-density-gradient case. By similar reasoning, the multidirectional three-dimensional distortions of the light path through the turbulent atmosphere would probably lead to estimates of position error no larger than those calculated for the constant-density-gradient case. Thus, the limits on the resolving power of the atmosphere, as seen from an aerospacecraft, are probably no greater for a vertical light ray than was estimated by equation (14).

Slanted Light-Ray Path

The nonvertical light-ray path is curved even without turbulence because of the radial density gradient in the atmosphere (ref. 11). Also the effects of the earth's curvature may have to be included. For this problem, a coordinate system (fig. 4) with origin at the earth's center is convenient. As in the first problem discussed, a linear density variation for a short distance δ normal to the light path is assumed at some particular instant with an exponential decay factor to account for the altitude $R - R_0$:

$$(n - 1) = k \frac{\rho}{\rho_{sl}} = k\left[1 + \left(\frac{\partial f}{\partial x}\right)_0 \delta\right] e^{-(R-R_0)/y_0} \tag{22}$$

The term $(\partial f/\partial x)_0$ is a constant, as in equation (4), and δ is a coordinate distance normal to the light path lying in the plane established by the light path and the earth-centered radius vectors. From equations (1) and (22) plus the observation that $\partial\delta/\partial r = -1$, the radius of curvature conforms to the relation

$$\frac{1}{r} = -\frac{\partial n}{\partial r} = \left\{k\left(\frac{\partial f}{\partial x}\right)_0 + k\left[1 + \left(\frac{\partial f}{\partial x}\right)_0 \delta\right] \frac{\sin \psi}{y_0}\right\} e^{-(R-R_0)/y_0} \tag{23}$$

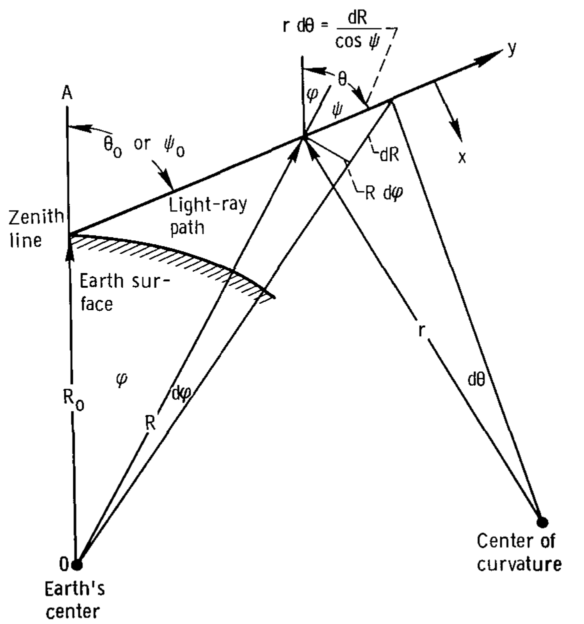

Figure 4. - Coordinate systems for slanted-ray derivations.

Let θ be the angle that the light-ray tangent makes to the zenith reference line OA. From figure 4,

$$\theta = \varphi + \psi \tag{24}$$

Also

$$r\ d\theta = \frac{dR}{\cos\psi} \tag{25}$$

and

$$\tan\psi = \frac{R\ d\varphi}{dR} \tag{26}$$

By elimination,

$$r\left(\frac{dR}{R}\tan\psi\right) + d\psi = \frac{dR}{\cos\psi} \tag{27}$$

Equations (23) and (27) give

$$\frac{d\psi}{\tan\psi} = \left[\frac{k}{y_0}e^{-(R-R_0)/y_0} - \frac{1}{R}\right]dR + \left[k\left(\frac{\partial f}{\partial x}\right)_0\left(\frac{1}{\sin\psi} + \frac{\delta}{y_0}\right)e^{-(R-R_0)/y_0}\right]dR \tag{28}$$

The extension δ of the light path from its nonturbulent position is small compared with y_0 so that δ/y_0 may be neglected compared with $1/\sin\psi$ giving

$$d\psi = \tan\psi\left[\frac{k}{y_0}e^{-(R-R_0)/y_0} - \frac{1}{R}\right]dR + \frac{k}{\cos\psi}\left(\frac{\partial f}{\partial x}\right)_0 e^{-(R-R_0)/y_0}\ dR \tag{29}$$

Similarly, from equations (24), (26), and (29),

$$d\theta = \tan\psi\,\frac{k}{y_0}e^{-(R-R_0)/y_0}\ dR + \frac{k}{\cos\psi}\left(\frac{\partial f}{\partial x}\right)_0 e^{-(R-R_0)/y_0}\ dR \tag{30}$$

Let

$$\psi = \psi_1 + \psi_2 \text{ and } \theta = \theta_1 + \theta_2 \tag{31}$$

where ψ_1 and θ_1 are solutions to equations (29) and (30) when $(\partial f/\partial x)_0$ equals zero. Then,

$$\frac{d\psi_1}{\tan \psi_1} = \left[\frac{k}{y_0} e^{-(R-R_0)/y_0} - \frac{1}{R}\right] dR \tag{32}$$

and

$$\frac{d\theta_1}{\tan \psi_1} = \frac{k}{y_0} e^{-(R-R_0)/y_0} dR \tag{33}$$

From equations (29) to (33),

$$d\theta_2 = d\psi_2 = \frac{k\left(\frac{\partial f}{\partial x}\right)_0}{\cos \psi_1} e^{-(R-R_0)/y_0} dR \tag{34}$$

Equation (34) is obtained by replacing $\tan \psi$ and $\cos \psi$ by $\tan \psi_1$ and $\cos \psi_1$, respectively, in equations (29) and (30). This replacement is permitted because ψ_2 and θ_2 are small, of order 3 seconds of arc or less. The approximation was checked numerically and found to produce negligible error in the values of $\theta_2 = \psi_2$.

Integration of equation (32) leads to the curved light path due to the radial density gradient in the atmosphere

$$\frac{R \sin \psi_1}{R_0 \sin \psi_0} = e^{k\left[1-e^{-(R-R_0)/y_0}\right]} \tag{35}$$

where R_0 is the radius of the earth, and $\psi_0 = \theta_0$ is the initial angle that the light ray from the surface makes with the zenith line (fig. 4). Elimination of ψ between equations (35) and (26) leads to the differential equation for the light path

$$\frac{d\varphi}{dR} = \frac{\sin \psi_0 g(R)}{R\left(1 - \sin^2\psi_0 g^2(R)\right)^{1/2}} \tag{36}$$

where

$$g(R) = \frac{R_0}{R} e^{-k\left[1-e^{-(R-R_0)/y_0}\right]} \tag{37}$$

The curvature of the light path due to the radial density gradient shifts the apparent position of the image, but this shift can be corrected. Hence, the radial density gradient without turbulence does not decrease the resolution of the atmosphere. Integration of equation (36) will therefore not be pursued.

The angle ψ_1 can be eliminated between equations (34) and (35). Simplifying approximations, however, are more interesting. The exponential factor

$$1 - e^{-(R-R_0)/y_0}$$

varies from 0 to 1. Because $k = 0.0003$, the second member of equation (35) differs from 1 only by a few ten-thousandths. Hence,

$$R \sin \psi_1 \approx R_0 \sin \psi_0 \tag{38}$$

This equation for a straight line may be rewritten as

$$\cos \psi_1 = \cos \psi_0 \left\{1 + \tan^2 \psi_0 \left[1 - \left(\frac{R_0}{R}\right)^2\right]\right\}^{1/2} \tag{39}$$

From equation (35) or (39), if ψ_0 is not too large, replacement of $\cos \psi_1$ by $\cos \psi_0$ in equation (34) would cause almost no error for altitudes below about 32 kilometers where

$$1 - \left(\frac{R_0}{R}\right)^2 \approx \frac{1}{100}$$

If a 5-percent error is allowed for $\cos \psi_1$ (the integrated error would be much less), the maximum allowed angle may be estimated as

$$\frac{\tan^2 \psi_0}{100} \leq 0.1$$

and

$$\psi_0 \leq 72^{\circ}$$

The useful satellite viewing angle would generally be less than this value. Above 32 kilometers, the contributions to ψ_2 are almost nil because of the exponential term. Hence, equation (34) may be further approximated as

$$d\psi_2 = \frac{k\left(\frac{\partial f}{\partial x}\right)_0}{\cos \psi_0} e^{-(R-R_0)/y_0} \, dR \tag{40}$$

and

$$\psi_2 = \theta_2 = \frac{k\left(\frac{\partial f}{\partial x}\right)_0 y_0}{\cos \psi_0} \left[1 - e^{-(R-R_0)/y_0}\right] \tag{41}$$

which reduces to equation (7) when $\psi_0 = \theta_0 \doteq 0$.

The completion of the estimate is simplified by switching to an x, y-coordinate system with y parallel to and x perpendicular to the slanted light path. Hence,

$$dy = \frac{dR}{\cos \psi_1} \tag{42}$$

and

$$dx = \theta_2 \, dy \tag{43}$$

(The slightly curved y-axis is thus the curved light path resulting from the radial density gradient.) Integration of equation (43) with the aid of equations (41) and (42) gives the positional error s' perpendicular to the light path looking up:

$$x = s' = \frac{k y_0^2 \left(\frac{\partial f}{\partial x}\right)_0}{\cos \psi_0} \left[\frac{y}{y_0} - \int_{R_0}^{R} \frac{e^{-(R-R_0)/y_0}}{\cos \psi_1} \, dR \right] \tag{44}$$

Because of the exponential, $\cos \psi_1$ may be replaced by $\cos \psi_0$ under the integral sign without serious error giving

$$s' = \frac{k y_0^2 \left(\frac{\partial f}{\partial x}\right)_0}{\cos \psi_0} \left\{ \frac{y}{y_0} - \frac{1}{\cos \psi_0} \left[1 - e^{-(R-R_0)/y_0} \right] \right\} \tag{45}$$

The projected error s along the circular orbit of the spacecraft is obtained by dividing s' by $\cos \psi_1$ or

$$s = \frac{k y_0^2 \left(\frac{\partial f}{\partial x}\right)_0}{\cos \psi_1 \cos \psi_0} \left\{ \frac{y}{y_0} - \frac{1}{\cos \psi_0} \left[1 - e^{-(R-R_0)/y_0} \right] \right\} \tag{46}$$

The approximation of equation (38) is used to obtain the value of y by integration of equation (42):

$$y = \left(R^2 - R_0^2 \sin^2 \psi_0 \right)^{1/2} - R \cos \psi_0 \tag{47}$$

Analogous to equation (10),

$$s_1' = \theta_2 y - s' \tag{48}$$

or, from equations (41) and (45),

$$s_1' = \frac{k \left(\frac{\partial f}{\partial x}\right)_0 y_0^2}{\cos \psi_0} \left\{ \frac{1}{\cos \psi_0} \left[1 - e^{-(R-R_0)/y_0} \right] - \frac{y}{y_0} e^{-(R-R_0)/y_0} \right\} \tag{49}$$

The projected error s_1 along the earth's surface is obtained by dividing s_1' by $\cos \psi_0$:

$$s_1 = \frac{k\left(\frac{\partial f}{\partial x}\right)_0 y_0^2}{\cos^2 \psi_0} \left\{ \frac{1}{\cos \psi_0} \left[1 - e^{-(R-R_0)/y_0}\right] - \frac{y}{y_0} e^{-(R-R_0)/y_0} \right\} \tag{50}$$

Equation (50) reduces to equation (11) when $\psi_0 = 0$, in which case $(R - R_0) = y$. Hence, the more general form of equation (14) (in cm) is

$$\text{Random error} \leq \frac{12}{\cos^2 \psi_0} \left\{ \frac{1}{\cos \psi_0} \left[1 - e^{-(R-R_0)/y_0}\right] - \frac{y}{y_0} e^{-(R-R_0)/y_0} \right\} \tag{51}$$

Even at values of ψ_0 as high as 72°, neglect of the exponential terms for altitudes above 50 kilometers produces only a few percent error.

If the angle ψ_0 and the altitude are not too great, a useful approximation for y may be obtained by series expansion of equation (47):

$$y \approx \frac{R - R_0}{\cos \psi_0} \left[1 - \frac{(R - R_0)}{2R_0} \tan^2 \psi_0 - \frac{(R - R_0)^2}{2R_0^2} \frac{\tan^2 \psi_0}{\cos^2 \psi_0} + \ldots \right] \tag{52}$$

Because of the exponential multiplier, only the first term need be used in equation (51). Hence, the precision (in cm) with which an aerospacecraft observer may view the ground through the transient atmosphere is as follows:

(1) In the plane established by the light ray and the zenith line,

$$\text{Random error} \leq \frac{12}{\cos^3 \psi_0} \left[1 - e^{-(R-R_0)/y_0} - \frac{(R - R_0)}{y_0} e^{-(R-R_0)/y_0}\right] \tag{53}$$

(2) Perpendicular to the plane established by the light ray and the zenith line,

$$\text{Random error} \leq \frac{12}{\cos^2 \psi_0} \left[1 - e^{-(R-R_0)/y_0} - \frac{(R - R_0)}{y_0} e^{-(R-R_0)/y_0}\right] \tag{54}$$

The light ray from the ground, of course, makes the angle $\theta_0 = \psi_0$ to the zenith. The altitude dependence of these random errors is plotted in figure 3.

Equation (54) was obtained by repeating the derivation for the case in which the density gradient is perpendicular to both the light ray and the radius vector R. The angle θ_2 is identical to that given by equation (41) as are the positional uncertainties s' (eq. (45)) and s_1' (eq. (49)). However, in this case $s' = s$ and $s_1' = s_1$ so that no additional $\cos \psi$ term is required in the denominator as was the case for equations (46), (50), and (51).

The approximation $y \approx (R - R_0)/\cos \psi_0$ may also be employed in equations (50) and (46) to give

$$s_1 = \frac{k\left(\frac{\partial f}{\partial x}\right)_0 y_0^2}{\cos^3 \psi_0}\left[1 - e^{-(R-R_0)/y_0} - \frac{(R - R_0)}{y_0} e^{-(R-R_0)/y_0}\right] \tag{55}$$

and

$$s = \frac{k\left(\frac{\partial f}{\partial x}\right)_0 y_0^2}{\cos^3 \psi_0}\left[\frac{(R - R_0)}{y_0} - 1 + e^{-(R-R_0)/y_0}\right] \tag{56}$$

Equation (55) is generally as valid as equations (50) and (51). The use of equation (56), however, must be restricted to altitudes below about 320 kilometers if the angle ψ_0 is as large as 45° ($2\frac{1}{2}$ percent error).

Observational Limits Due to Other Causes

The diffraction limited resolution of an optical system onboard an aerospacecraft used for ground observation need be no greater than the values of equations (53) and (54) (assuming 3 sec of arc uncertainty for a twinkling star). The telescope objective for best resolution, as viewed from a satellite at an altitude of 320 kilometers, requires a diameter of about 1.6 meters to resolve a 12-centimeter radius, or one-half this value if the diameter rather than the radius of the fuzziness due to the atmosphere is chosen. For the same viewing accuracy, an aerospacecraft flying at an altitude of 32 kilometers would require a telescope diameter one-tenth as large. The limiting capability of both to see the ground is essentially the same (within 9 percent of the error, see fig. 3). Both have the same cloud cover and contrast limits. The high-flying aerospacecraft would have the advantage over the satellite of being able to use smaller and perhaps cheaper optical equipment. Local (near the aircraft) aerodynamic disturbances might arise, however, to decrease the resolution. The satellite has a vastly superior flight endurance capability.

The satellite must, of course, travel at a speed of about 7.6 kilometers per second to stay in orbit. To achieve the resolution discussed heretofore would require either extremely short exposure times or motion compensation techniques to follow an object on the ground. If television were employed, the line spacing of the camera would have to be considered also.

The question might be raised as to whether sufficient illumination for observation exists. During the daytime, the visible light transmission coefficient through the entire atmosphere is about 85 percent at the zenith. Viewing the zenith through the entire atmosphere is about equivalent to looking at an object located horizontally on the surface about 8.5 kilometers away if the air is clear.

The observational capability would also be useful at night. According to reference 12, the unaided human eye requires at least 2.5×10^{-9} erg per second to detect a point light source. A 1-watt light bulb with 1-percent efficiency should therefore be observable from a 320-kilometer altitude with a 30-centimeter-diameter telescope. Clouds, smog, smoke, dust, and haze in the atmosphere would of course decrease the light available to the observer.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio September 26, 1968.

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